

DEVELOPING A NEW DATA ENVELOPMENT ANALYSIS MODEL FOR CUSTOMER VALUE ANALYSIS

MAHDI MAHDILOO, ABDOLLAH NOORIZADEH AND REZA FARZIPOOR SAEN¹

Department of Industrial Management, Faculty of Management and Accounting
Islamic Azad University-Karaj Branch, P. O. Box: 31485-313, Karaj, Iran
Tel: 0098 (261) 4418144-6
Fax: 0098 (261) 4418156

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ABSTRACT. This paper proposes an application of data envelopment analysis (DEA) to measure the value of customers. In order to distinguish between expectations and needs of profitable and unprofitable customers and to allocate marketing investments among them, customers are compared with each other and ranked in a customer value pyramid. To this end, we use a combination of the Banker, Charnes and Cooper (BCC) model [3], assurance region (AR) model, and cross-efficiency evaluation. A numerical example demonstrates the application of the proposed model in an Iranian manufacturing company.

1. Introduction. Determining the profitability of customers and ranking them is an issue that has been frequently studied in the past decades. Customers are the most important assets of companies and companies should view customer relationships as mutually beneficial exchanges [14] and opportunities that need to be managed [4]. In order to manage customer relationships, managers need to have a good relationship marketing system. Relationship marketing defines a strategy of approaching prioritized customers. In relationship marketing the effective factor in knowing and establishing long-term relationships with customers is the added value amount gained by the company. Therefore, the prioritizing of customers is one of the most important activities in relationship marketing. Prioritizing of customers and classifying them into different clusters aim at improving the performance of the marketing strategies and increasing the company's market share. As Garland [12] addresses, while companies may want to treat all customers with superior service, they find it neither practical nor profitable to meet all customers' expectations. Therefore, it is necessary to use the limited marketing resources to target those customers that generate the largest profits for the company.

After distinguishing among different types of customers, it is necessary to focus on requirements and expectations of the most profitable customers in order to retain them. Since it costs five times more to acquire a new customer than to retain an existing one [22], customer retention is a more effective business strategy than continuously trying to acquire new customers in order to replace the defecting ones

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¹Corresponding author

[2]. At the same time, customers that are alike those in the least profitable segments, can be avoided, or at least customer acquisition investments in those segments can be reduced [33].

Reinartz and Kumar [25] proposed a framework for customer segmentation. Their customer segmentation concept is a 2×2 matrix based upon profitability and customer tenure. This produces four different types of customer classification; butterflies, true friends, strangers and barnacles. Recognizing the importance of customers' profits, Zeithaml et al. [37] worked on the customer pyramid concept. They clearly focused on the "top of the pyramid" those consumers with the highest customer lifetime value (CLV). By dividing the customer pyramid into four sections called customer profitability tiers, they identified the "best" most profitable customers, and labeled them as "Platinum" and "Gold". In contrast, those with low and very low profits, earn the value labels "Iron and Lead".

As Gönöl and Shi [13] discussed, the scenario in a typical model for customers scoring starts with research analyst developing a customer response model, e.g. estimating a multiple regression or a logit/probit equation where the left-hand side is a discrete dependent variable for purchase/nonpurchase. The independent variables are typically composed of purchase history variables, (usually characterized by the RFM triplet (recency, frequency, monetary value of the purchase amount), where recency stands for elapsed time since the last purchase, frequency refers to the number of purchases in the past or proportion of purchases over a period of time, and monetary value is the amount spent so far or average amount per purchase so far. Colombo and Jiang [7] focused on a stochastic RFM model to determine a ranking of marketing research customers in terms of their expected contribution or lifetime value. Aaker et al. [1] used a linear statistical method such as logistic regression to model response based on a test of a random sample of customers from the complete list. Fader et al. [11] presented a model that links the well-known RFM paradigm with CLV. The stochastic model used in their paper is based on the Pareto/NBD framework to capture the flow of transactions over time and a gamma-gamma submodel for spending per transaction. Deichmann et al. [9] investigated the use of a multiple adaptive regression splines (MARS), together with logistic regression in the context of modeling direct response. In their study they showed that the MARS model outperforms the logistic model in general. Moutinho et al. [21] predicted bank customers' responses using artificial neural networks (ANN). Kim et al. [18] proposed an approach that uses ANNs guided by genetic algorithms (GAs) to the prediction of households interested in purchasing an insurance policy for recreational vehicles (RVs). The trained model is then used to rank the potential customers in descending order of purchase probability. Huanga et al. [17] applied a learning method based on statistical learning theory, support vector machines (SVM), together with a frequently used method, backpropagation neural networks (BNN), to solve the problem of customers' credit rating prediction.

Hansen and Salamon [16] argue that often a simple logistic regression predicts better than an ANN. One major reason is that an ANN model has to be built with great care. In particular, its performance is sensitive to its complexity, determined by the number of synapses or weight parameters. If a network is more complex than the problem at hand or the available data set required, then the network learns not only the underlying function but also the noise peculiar to the finite training data set. In addition, as Ha et al. [15] state, ANN is a black box that sheds little light on what is going on inside the model. The input variables are combined in a

complicated, nonlinear way to produce outputs. Those marketers who would like to understand how individual predictor variables influence the target and how they interact might be baffled by the ANN model's inability to provide any insight in that regard.

In this paper, data envelopment analysis (DEA), as a nonparametric and multiple criteria decision making tool, is used to evaluate customers' value. DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) [5] and it is a linear-programming-based methodology that uses multiple inputs and multiple outputs to calculate efficiency scores. The efficiency score for each decision making unit (DMU) is defined as a weighted sum of outputs divided by a weighted sum of inputs, where all efficiencies are restricted to a range from 0 to 1. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights [20].

Wong and Wong [35] listed some of the features of DEA as below which motivated us to use it as a tool for prioritizing the customers.

- DEA is an effective tool for evaluating the relative efficiency of DMUs in the presence of multiple performance measures.
- DEA utilizes the concept of efficient frontier as a measure for performance evaluation. This ability of DEA identifies appropriate reference DMUs and includes easily interpretable efficiency parameters. These parameters are helpful in setting realistic and attainable standards or benchmarks. Therefore, it enables us to benchmark the performance of good customers. Benchmarking is a process of defining valid measures of performance comparison among DMUs. The efficient frontier used in DEA serves appropriately as an empirical standard of excellence. Hence, DEA is suitable to be regarded as the benchmarking tool.
- DEA is able to address the complexity arising from the lack of a common scale of measurement.
- In DEA, one does not need to assume a priori the existence of a particular production function for weighting and aggregating inputs or outputs.
- The objectivity stemming from DEA weighting variables during the optimization procedure frees the analysis from subjective estimates.

1.1. Comparison of DEA with regression-type models. In this subsection, we compare DEA with regression-type models graphically and show how DEA outperforms regression-type models. Assume that there are eight customers who are supposed to be prioritized with regard to two factors: their profitability and their average payment period. It is preferred by the company to have customers with lower payment period and higher profitability. Therefore, profitability of customers is considered as an output and is shown on the vertical axis of Fig. 1. However, average payment period is treated as an input and is shown on the horizontal axis. DEA identifies customer *B* as the most efficient customer and *F* as the most inefficient. Each point can be connected by drawing a line to the origin. The highest slope is attained by the line from the origin passing through *B*. This line is called the "efficient frontier". Notice that this frontier includes at least one point and all points are therefore on or below this line.

Given these data, one might be tempted to draw a statistical regression line fitted to them. The dotted line in Fig. 1 shows the regression line passing through the origin. This line, as normally determined in statistics, goes through the middle of these data points and we can define the points above it as excellent and the points below it as inferior or unsatisfactory. One can measure the degree of excellence or inferiority of these data points by the magnitude of the deviation from the thus fitted line. On the other hand, the frontier line designates the performance of the best customer *B* and measures the efficiency of other customers by deviations from it. There thus exists a fundamental difference between regression analysis and DEA. The former reflects “average” or “central tendency” behavior of the observations while the latter deals with best performance and evaluates all performances by deviations from the frontier line [8].

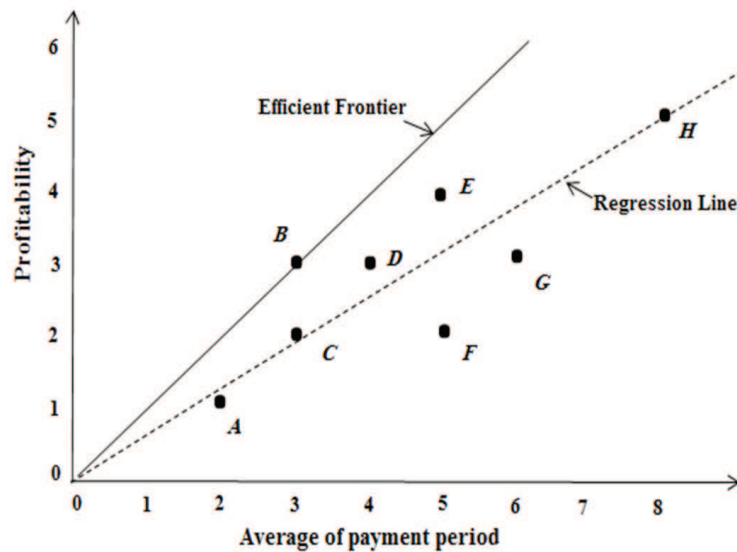


FIGURE 1. Regression line vs. frontier line

With the features and inherent characteristics of DEA discussed above, DEA is justified to be used as a prioritizing tool of customers.

The objective of this paper is to propose an assurance region-BCC (AR-BCC) DEA model that can deal with undesirable outputs. In order to have a complete ranking of customers, the proposed model is used in peer-appraisal instead of self-appraisal. Using the results of the proposed DEA model and based on the work of Zeithaml et al. [37], a customer pyramid is constructed.

To the best of knowledge of authors, there is not any reference that uses DEA to evaluate the value of customers. Some of the contributions of this paper are listed below:

- This is the first time that DEA is used to evaluate customers' value.
- Although in business-to-business marketing, the emphasis is on relationships rather than transactions, as Yu and Cai [36] declares, in most of the studies customers are treated as black boxes, only their final response or intention to buy is being assessed, and those only in simple dichotomous terms: to

response or not to response; to buy or not to buy. In our study customers are not treated as black boxes and the quality of their transactions is also considered. We focus not only on customers’ decisions to buy or not to buy, but also on their performance during transaction period. That is, in addition to customers’ profitability, we consider their credit, payments on due date, average payment period and purchase return, to indicate which ones are good customers and which ones are not. Therefore, the proposed model evaluates customers’ value in a multiple criteria context.

- Accounting managers subtract sales return of customers ² from their gross sales revenue to obtain net sales revenue and reveal it in the income statements [30]. However, accounting managers do not take into account other costs such as energy costs, transaction costs, and other hidden costs which those customers have imposed to company. Due to the fact that these costs can be significant and there is lack of market prices for them, it is extremely difficult to estimate them. Therefore, to overcome these measurement and evaluation difficulties, purchase returns of customers is treated as undesirable output in the proposed DEA model.
- Aggressive formulation of AR-BCC model which considers undesirable outputs is developed to evaluate the peer-appraisal value of customers instead of their self-appraisal.

This paper proceeds as follows. Section 2, introduces the model which analyzes customers value. Numerical example and concluding remarks are discussed in Sections 3 and 4, respectively.

2. Proposed model. In this paper, we use DEA to estimate customers’ value to a company. Banker et al. [3] proposed BCC model, Model (1), as an extension of CCR model to accommodate technologies that exhibit variable returns to scale. The nomenclatures used in this paper are presented in Table 1.

The input-oriented BCC model evaluates the relative value of customer under investigation (DMU_d) ($d = 1, \dots, n$) by solving the following linear program.

$$\begin{aligned}
 &Max h_A = \sum_{r=1}^k \mu_r y_{rd} - w_d \\
 &s.t. \\
 &\sum_{i=1}^m v_i x_{id} = 1, \\
 &\sum_{r=1}^k \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} - w_d = 0, \quad j = 1, \dots, n \\
 &v_i \geq 0, \quad i = 1, 2, \dots, m, \\
 &\mu_r \geq 0, \quad r = 1, 2, \dots, k, \\
 &w_d \text{ free in sign.}
 \end{aligned} \tag{1}$$

Model (2) shows the dual (envelopment form) of Model (1)

² Since the purpose of this study is to analyze the performance of customers, we consider “sales returns” from the perspective of customers and therefore use “purchase returns” term instead of “sales returns”.

$$\begin{aligned}
& \text{Min } h_B = \theta \\
& \text{s.t.} \\
& \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{id}, \quad i = 1, 2, \dots, m, \\
& \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{rd}, \quad r = 1, 2, \dots, k, \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j, s_i^-, s_r^+ \geq 0.
\end{aligned} \tag{2}$$

2.1. Undesirable outputs. As mentioned earlier, purchase return of customers is one of the criteria used in this paper that can be treated as an undesirable output. Seiford and Zhu [28] proposed Model (3) as an output-oriented BCC model that can treat undesirable outputs. To consider undesirable outputs in an envelopment (dual) form of BCC model, they suggested a linear monotone decreasing transformation, $\bar{y}_{sj}^b = -y_{sj}^b + v > 0$, where v is a proper translation vector that makes $\bar{y}_{sj}^b > 0$.

$$\begin{aligned}
& \text{Max } h_C = \theta \\
& \text{s.t.} \\
& \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{id}, \quad i = 1, 2, \dots, m, \\
& \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \theta y_{rd}, \quad r = 1, 2, \dots, k, \\
& \sum_{j=1}^n \lambda_j \bar{y}_{sj}^b - s_s^+ = \theta \bar{y}_{sd}^b, \quad s = k + 1, \dots, p \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j, s_i^-, s_r^+, s_s^+ = 0.
\end{aligned} \tag{3}$$

Table 2 presents the data set for a simple hypothetical numerical example involving 10 DMUs, with a single input, a desirable output and an undesirable output which reveals a problem in Model (3). Notice that this problem occurs because of the arbitrariness of the v . That is, when we translate the original data of undesirable output with different amounts of v and run Model (3), the classification of the DMUs as weak-efficient or inefficient remains, but the efficiency score of each inefficient unit is distorted. The efficiency scores defined in Model (3) with $v = 15$ and $v = 20$ are reported in Table 2. It can be easily seen that the results obtained by $v = 15$ and $v = 20$ are different from each other which reduces the validity of the model.

As Zhu and Cook [38] discussed, the translation invariance property allows the envelopment form of many DEA models to translate inputs or outputs data without any difference between the results of translated data and original data. The envelopment form of the input (output)-oriented BCC model is translation invariant with respect only to outputs (inputs). For more details please see Theorem 1.

This theorem means that, for example, we can deal with any output variable in the input-oriented BCC model, even if all its data are translated. It should also be noted that the BCC model is variable returns to scale (VRS) model, in contrast to the CCR model, which exhibits constant returns to scale (CRS). In fact, being a VRS model is the key to satisfy translation invariance or, in other words, the convexity constraint (the sum of the intensity variables equals 1, i.e., $\sum_{j=1}^n \lambda_j = 1$) is the clue. Therefore, the strategy of Seiford and Zhu [28] to change undesirable outputs to desirable outputs has a limitation; i.e. before using any model, the translation invariance property of the model should be viewed first.

Fig. 2 demonstrates lack of translation invariance property of Models (2) and (3), graphically.

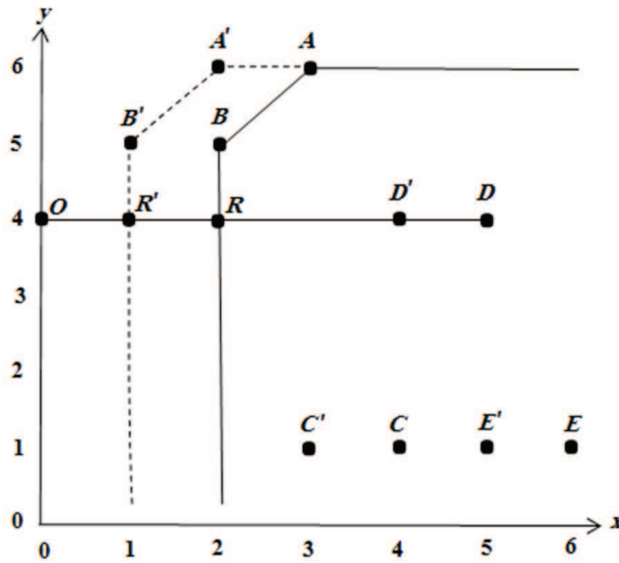


FIGURE 2. Translation in the BCC model

In this figure, DMU_D has the input-oriented BCC efficiency OR/OD which is the distance of DMU_D from the efficiency frontier constructed by efficient units A and B . Since OR/OD is equal to the objective function of Model (2), so $OR/OD = \theta = 2/5$. This ratio is not invariant when we translate input values by deducting a unity from them. Now, efficiency frontier shifts to the left and input-oriented BCC efficiency of $DMU_{D'}$, DMU_D after translation, becomes $OR'/OD' = \theta' = 1/4$ which is the distance of $DMU_{D'}$ from the efficiency frontier constructed by efficient units A' and B' . Since $\theta \neq \theta'$, the input-oriented BCC model is not translation invariant with respect to inputs.

Theorem 2.1. *The input-oriented BCC model is not translation invariant with respect to inputs.*

Proof. Let us translate the data set (X, Y) by introducing arbitrary constants $(\Phi_i: i=1, \dots, m)$ and $(\Psi_r: r=1, \dots, k)$ to obtain new data.

$$x'_{ij} = x_{ij} - \Phi_i \tag{4}$$

$$y'_{ij} = y_{rj} - \Psi_r \tag{5}$$

To show that this model is not translation invariant, under this arbitrary translation, we observe that x values in the constraint $\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{id}$ become

$$\sum_{j=1}^n \lambda_j (x'_{ij} + \Phi_i) + s_i^- = \theta' (x'_{id} + \Phi_i).$$

With regard to (4), $\sum_{j=1}^n \lambda_j (x'_{ij} + \Phi_i) + s_i^- = \theta' (x'_{id} + \Phi_i)$ becomes

$$\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta' x_{id}$$

Since $\theta \neq \theta'$, so $\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta' x_{id} \neq \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{id}$. Therefore the input-oriented BCC model is not translation invariant with respect to inputs. \square

Notice that similar process can be done to show that the output-oriented BCC model is not translation invariant with respect to outputs. In addition, similar process can be repeated for demonstrating lack of translation invariance property of Model (3).

Korhonen and Luptacik [19] proposed a CCR model with undesirable outputs to measure the efficiency of power plants. This model treats undesirable outputs as inputs and do not suffer from the problem discussed above. We formulate the BCC version of their model as below.

$$\begin{aligned} \text{Max } h_D &= \sum_{r=1}^k \mu_r^g y_{rd}^g - w_d \\ \text{s.t.} \\ \sum_{i=1}^m v_i x_{id} + \sum_{s=k+1}^p \mu_s^b y_{sd}^b &= 1, \\ \sum_{r=1}^k \mu_r^g y_{rj}^g - \left(\sum_{i=1}^m v_i x_{ij} + \sum_{s=k+1}^p \mu_s^b y_{sj}^b \right) - w_d &= 0, \quad j = 1, 2, \dots, n, \\ \mu_r^g &\geq 0, \quad r = 1, 2, \dots, k, \\ v_i &\geq 0, \quad i = 1, 2, \dots, m, \\ \mu_s^b &\geq 0, \quad s = k + 1, \dots, p, \\ w_d &\text{ free in sign.} \end{aligned} \tag{6}$$

Therefore, Model (6) is a BCC model which can consider undesirable outputs.

2.2. Weight restrictions. In many applications, it may be reasonable to set the lower and upper bounds of the ratio of virtual weights of inputs and undesirable outputs as below.

$$\alpha_i \leq \frac{v_i}{v_{i+1}} \leq \beta_i, \tag{7}$$

$$\phi_i \leq \frac{\mu_s^b}{v_i} \leq \psi_i, \tag{8}$$

Similarly, for outputs, we can impose the lower (upper) bounds on the ratio of the virtual weights of outputs r to $r+1$ as follows and use the assurance region model

of Thompson et al (1986);

$$\tau_r \leq \frac{\mu_r^g}{\mu_{r+1}^g} \leq \gamma_r, \tag{9}$$

The Greek letters $(\alpha_i, \beta_i, \phi_i, \psi_i, \tau_r, \gamma_r)$ are user specified constants to reflect value judgments the decision maker (DM) wishes to incorporate in the assessment. They may relate to the perceived importance or worth of input and output factors.

Now we convert the model of Korhonen and Luptacik [19] to a new BCC model with weight restrictions.

$$\begin{aligned}
 E_{dd} = \text{Max } h_E &= \sum_{r=1}^k \mu_r^g y_{rd}^g - w_d \\
 \text{s.t.} \\
 \sum_{i=1}^m v_i x_{id} + \sum_{s=k+1}^p \mu_s^b y_{sd}^b &= 1, \\
 \sum_{r=1}^k \mu_r^g y_{rj}^g - \left(\sum_{i=1}^m v_i x_{ij} + \sum_{s=k+1}^p \mu_s^b y_{sj}^b \right) - w_d &\leq 0, \quad j = 1, 2, \dots, n, \\
 \alpha_i v_{i+1} &\leq v_i, \quad i = 1, 2, \dots, m, \\
 v_i &\leq \beta_i v_{i+1}, \quad i = 1, 2, \dots, m, \\
 \phi_i v_i &\leq \mu_s^b, \quad i = 1, 2, \dots, m, s = k + 1, \dots, p, \\
 \mu_s^b &\leq \psi_i v_i, \quad i = 1, 2, \dots, m, s = k + 1, \dots, p, \\
 \tau_r \mu_{r+1}^g &\leq \mu_r^g, \quad r = 1, 2, \dots, k, \\
 \mu_r^g &\leq \gamma_r \mu_{r+1}^g, \quad r = 1, 2, \dots, k, \\
 \mu_r^g &\geq 0, \quad r = 1, 2, \dots, k, \\
 v_i &\geq 0, \quad i = 1, 2, \dots, m, \\
 \mu_s^b &\geq 0, \quad s = k + 1, \dots, p, \\
 w_d &\text{ free in sign.}
 \end{aligned} \tag{10}$$

Therefore, Model (10) is a BCC model which in addition to undesirable outputs has weight restrictions.

To determine benchmarks of each inefficient DMU, the envelopment from of Model (10) is written as follow:

$$\begin{aligned}
 \text{Min } \theta \\
 \theta x_{id} - \sum_{i=1}^m \lambda_j x_{ij} + \alpha_i \pi_1 - \pi_1 + \pi_2 - \beta_i \pi_2 &\geq 0, \quad j = 1, \dots, n \\
 \theta y_{sd}^b - \sum_{s=k+1}^p \lambda_j y_{sj}^b + \phi_i \eta_1 - \eta_1 + \eta_2 - \psi_i \eta_2 &\geq 0, \quad j = 1, \dots, n \\
 \sum_{r=1}^k \lambda_j y_{rj}^g + \tau_r \xi_1 - \xi_1 + \xi_2 - \gamma_r \xi_2 &\geq y_{rd}^g, \quad j = 1, \dots, n \\
 \sum_{j=1}^n \lambda_j &= 1 \\
 \pi_1, \pi_2, \eta_1, \eta_2, \xi_1, \xi_2, v_i, \mu_r^g, \mu_s^b &\geq 0.
 \end{aligned} \tag{11}$$

2.3. Using analytic hierarchy process (AHP) to determine the lower and upper bounds of weights. To determine the lower and upper bounds of weights defined in (7), (8) and (9), we use AHP technique developed by Saaty [26] as a multiple criteria decision making tool. In this technique the weights can be assigned to each criterion through a process called pairwise comparison. Comparisons can be done by verbal comparison. Decision-makers compare criteria for their relative importance using words such as Equal, Moderate, Strong, Very Strong, and Extreme. Thus, one of the questions that one may ask when using pairwise comparison is ‘how important is each factor for analyzing customer value?’ The answer can be ‘Equal’, ‘Moderate’, etc. The verbal responses are then quantified and translated into a score through the nine-point scale shown in Table 3.

In a similar method, Seiford and Zhu [27] used DEA to investigate excesses and deficits in Chinese industrial productivity for the years (1953-1990). They used Delphi and AHP techniques for imposing weights to DEA model and used geometric mean to combine experts’ individual opinions to derive weights from multiplicative pairwise comparisons. Seiford and Zhu [27] used the average of the weights, however due to the variances, these averages cannot reflect all the experts’ opinions. To avoid this drawback, we determine the union of all the weights derived from the opinions of group of experts as upper and lower bounds in DEA constraints.

2.4. Cross-efficiency evaluation. Efficiency scores calculated by Models (10) and (11) can not rank all DMUs and there may exist lack of discrimination among efficient DMUs. To overcome this problem, the cross-efficiency method introduced by Sexton et al. [29] is used. The cross-efficiency method was developed as a DEA extension tool that can be utilized to identify best performing DMUs and to rank DMUs. The main idea of cross-efficiency is to use DEA in a peer evaluation instead of a self evaluation mode.

For each DMU_d ($d=1, \dots, n$), in Model (10), we can obtain a set of optimal weights (multipliers) ($\mu_r^{*g}, \mu_s^{*b}, v_i^*$). Using these set of weights, the cross-efficiency for any DMU_j ($j=1, \dots, n$), is then calculated as:

$$E_{dj} = \frac{\sum_{r=1}^k \mu_{rd}^{*g} y_{rj}^g - w_d}{\sum_{i=1}^m v_{id}^* x_{ij} + \sum_{s=k+1}^p \mu_{sd}^{*b} y_{sj}^b}, \quad d, j = 1, 2, \dots, n \quad (12)$$

where E_{dj} shows the relative efficiency of DMU_j with optimal weights for inputs and outputs of DMU_d. One could compute the average of the efficiencies in each column to get a measure whether the DMUs associated with the column are rated by the rest of the DMUs. Good operating practices are more likely to be exhibited by relatively efficient DMUs offering high average efficiencies in their associated columns in the cross-efficiency matrix. Since Model (10) will be run n times for n DMUs, respectively, each DMU will get n efficiency scores, which construct a $n \times n$ matrix, called cross-efficiency matrix. For DMU_j ($j=1, \dots, n$), the average of all E_{dj} ($d=1, \dots, n$), namely

$$\bar{E}_j = \frac{1}{n} \sum_{d=1}^n E_{dj} \quad (13)$$

can be used as an efficiency measure for DMU_j, and will be referred to as the cross-efficiency score for DMU_j.

The non-uniqueness of the DEA optimal weights possibly reduces the usefulness of cross-efficiency. To overcome this problem, Doyle and Green [10] suggested

the use of aggressive and benevolent cross-evaluation. A cross-efficiency is aggressive/benevolent in the sense that it selects a set of weights which not only maximize the efficiency of DMU_d , but also minimize/maximize the efficiencies of all other DMUs in some sense.

At this juncture, the new model which considers AR, VRS, and undesirable outputs in the context of aggressive cross-efficiency is proposed. This model is based on the proposed Model (10). Note that the benevolent formulation has the same set of constraints except that the objective function is maximized.

$$\begin{aligned}
 \text{Min } h_F &= \mu_r^g \sum_{j \neq d} y_{rj}^g - w_d \\
 \text{s.t.} & \\
 \sum_{r=1}^k \mu_r^g y_{rj}^g - \left(\sum_{i=1}^m v_i x_{ij} + \sum_{s=k+1}^p \mu_s^b y_{sj}^b \right) - w_d &\leq 0, j \neq d, \\
 v_i \sum_{j \neq d} x_{ij} + \mu_s^b \sum_{j \neq d} y_{sj}^b &= 1, \\
 \sum_{r=1}^k \mu_r^g y_{rd}^g - E_{dd} \left(\sum_{i=1}^m v_i x_{id} + \sum_{s=k+1}^p \mu_s^b y_{sd}^b \right) - w_d &= 0, \\
 \alpha_i v_{i+1} &\leq v_i, i = 1, 2, \dots, m, \\
 v_i &\leq \beta_i v_{i+1}, i = 1, 2, \dots, m, \\
 \phi_i v_i &\leq \mu_s^b, \quad i = 1, 2, \dots, m, s = k+1, \dots, p, \\
 \mu_s^b &\leq \psi_i v_i, \quad i = 1, 2, \dots, m, s = k+1, \dots, p, \\
 \tau_r \mu_{r+1}^g &\leq \mu_r^g, \quad r = 1, 2, \dots, k, \\
 \mu_r^g &\leq \gamma_r \mu_{r+1}^g, \quad r = 1, 2, \dots, k, \\
 \mu_r^g &\geq 0, \quad r = 1, 2, \dots, k, \\
 v_i &\geq 0, \quad i = 1, 2, \dots, m, \\
 \mu_s^b &\geq 0, \quad s = k+1, \dots, p, \\
 w_d &\text{ free in sign.}
 \end{aligned} \tag{14}$$

where E_{dd} is the efficiency of DMU_d obtained by Model (10). Fig. 3 depicts the above mentioned discussions graphically.

3. Numerical example. In order to illustrate the usage of the newly developed model, the evaluation of customers' value in an Iranian company (Achachi Co.) is presented. These customers are wholesalers of company. Achachi was established in 1979 and works in the field of food industry. The mission of this company is to satisfy customers' needs for different kinds of chocolate, cacao, chocolate cacao, jelly, chocolate jelly, juice powder, etc. The data set for this study consists of annual observation for the year 2008. Five criteria proposed in Table 4 have been used as measures for customers' value evaluation. Average payment period is as an input with smaller value being better; credit, profitability, and payments on due date are desirable outputs with larger values being better and purchase return is an undesirable output with smaller value being better. Table 5 depicts data set to evaluate the value of 37 customers (DMUs).

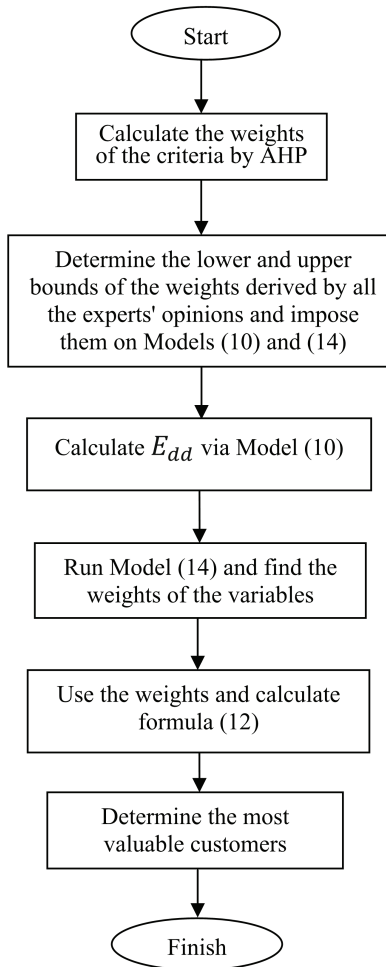


FIGURE 3. Flow chart of the proposed model

As Wang et al. [34] discussed, in order to eliminate the impacts of units of measurement all inputs and outputs should be normalized. Table 6 shows the normalized inputs and outputs. The normalization of the inputs and outputs can be performed by dividing the inputs and outputs values to the summation of that particular input or output.

To find upper and lower bounds of weights and to restrict weights of proposed model, the priorities of four DMs are compared through pairwise comparison matrix using Saaty's 1-9 scale. Tables 7 to 10 show comparisons of four DMs to find upper and lower bounds of outputs' weights. Tables 11 to 14 show pairwise comparison matrices for the input and undesirable output determined by four DMs.

After inputting pairwise comparisons into Expert Choice software, the weights for inputs and outputs, with regard to each DM's opinion, were obtained. Tables 15 and 16 depict the results.

Inconsistency ratios have been depicted in the last columns of Tables 15 and 16 which are within acceptable ratio of 0.1, as it is recommended by Saaty [26].

As Sueyoshi et al. [31] describe, the concept of consistency assesses the quality of judgments made during a series of pairwise comparison. The inconsistency indices measure the degree of inconsistency in a series of pairwise comparison. It is useful to examine how the judgments are consistent with each other. If the ratio is greater than 0.1, it indicates the status of inconsistency in the pairwise judgment. In the opposite case, the pairwise comparisons are reasonable and the AHP process can continue with the synthesis of computations.

Now, to incorporate the weights of inputs and outputs into the Models (10) and (14), the results of Tables 15 and 16 are unionized.

$$DM1\# \frac{\mu_2^g}{\mu_1^g} = \frac{0.174}{0.634} = 0.27445$$

$$DM2\# \frac{\mu_2^g}{\mu_1^g} = \frac{0.592}{0.333} = 1.77778$$

$$DM3\# \frac{\mu_2^g}{\mu_1^g} = \frac{0.23}{0.648} = 0.35494$$

$$DM4\# \frac{\mu_2^g}{\mu_1^g} = \frac{0.528}{0.333} = 1.58559$$

Therefore, we have

$$0.274 \leq \frac{\mu_2^g}{\mu_1^g} \leq 1.778$$

where the above restriction is the union of four experts' opinions on the importance of profitability of customers relative to their credit. Notice that, in order to impose this weight restriction into the Models (10) and (14) it should be divided into following two parts:

$$0.274\mu_1^g \leq \mu_2^g \text{ and } \mu_2^g \leq 1.778\mu_1^g$$

By repeating the above process for other criteria we have:

$$0.188 \leq \frac{\mu_3^g}{\mu_1^g} \leq 0.420$$

$$0.188 \leq \frac{\mu_3^g}{\mu_2^g} \leq 1.103$$

$$0.5 \leq \frac{\mu_1^b}{v_1} \leq 3$$

Table 17 shows the results of evaluation derived by different approaches. Column 2 of this table depicts amount of customers' profitability. Note that the evaluation of customers based on their profitability is a one-dimensional comparison which does not consider other criteria introduced in Table 4. According to this measure of customers' profitability, customer #2 is the best customer. Column 3 of this table shows the result of evaluation after running Model (10). As mentioned earlier, Model (10) not only considers profitability of customers, but also takes into account other criteria. In this evaluation each customer seeks to maximize its efficiency score by choosing a set of optimal weights for all inputs and outputs that are between the upper and lower bounds imposed by four DMs. This time the best customers are 1, 2, 15, and 33 with their efficiency scores equal to unity.

Benchmarks (peer groups) of each inefficient customer are one of the most important information of DEA that can be derived by Model (11). These benchmarks

are shown in column 4. For example consider customer #3 which is an inefficient customer. Model (11) selected customer #15 as the benchmark customer for this customer. It means that the managers of the company should try to encourage customer #3 to decrease its average payment period from 15 to 5.80 and purchase returns from 2661 to 211. As well, the seller company should try to increase this customer's profitability from 43152 to 50876. By doing this, customer #3 can reach its benchmark, customer #15, and be efficient.³

As you can see, Model (10) cannot give a complete ranking of customers and there are ties among efficient customers 1, 2, 15, and 33 with a relative efficiency score of 1. To overcome this problem, cross-efficiency approach is used and the final cross-efficiency scores of customers are depicted in the last column of Table 17. In this way, all the efficient customers are ranked. To derive the cross-efficiency matrix, Model (14) has been used. Under a cross evaluation, when the customer has chosen weighting system which all other customers have used, the efficiency score given to each customer is then used to form a cross-efficiency matrix. Once the matrix is filled, each customer has not only its own self evaluation but also the peer evaluations it has received via the other customers in the sample. The average among self and peer evaluations represents a cross-efficiency score of customer.

Optimal values derived by Model (14) and the cross-efficiency matrix are shown in Tables 18 and 19 respectively. Table 20 shows ranking results determined by profitability of customers and cross-efficiency scores.

To highlight the importance of the proposed model and its difference from the classical measure (profitability of customers), a Spearman correlation analysis between their results is shown in Table 21. Note that the data after ranking is an ordinal scale type, so it is suitable to use the Spearman Rank-order correlation [6]. Since correlation coefficient between the results of two approaches, at significant level of 0.01, is 0.026 there is a lack of similarity between ranking results. Therefore, applying the model proposed in this paper is necessary. This difference is only due to the multiple-criteria nature of DEA, while the profitability of customers is a one-dimensional measure.

Now, we use the concept of customer pyramid proposed by Zeithaml et al. [37], and classify customers into four groups as Platinum, Gold, Iron, and Lead (please see Fig. 4).

As Pitta et al. [23] describes, the Platinum tier represents the company's most profitable customers. They are often heavy users of a product or service and not overly price sensitive. The other valuable tier, the Gold tier is still attractive. It differs from the Platinum tier by lower but still good profitability levels. Also their commitment to the firm is lower than that of the Platinum tier. The two less attractive tiers, Iron and Lead represent much lower profit potential than others. The spending levels, loyalty, and profitability of Iron tier customers are not substantial enough for special treatment. In contrast, Lead tier customers represent losses to the company. They tend to cost more than they generate. They may demand more service than they are merit given their spending and profitability.

As Van Raaij [33] addresses, 1% of most valuable customers are labeled as Platinum and 4%, 15% and 80% of them are labeled as Gold, Iron, and Lead, respectively. As it can be seen, customer 1 is a Platinum customer and customer 15 is a Gold customer. Other customers are Iron and Lead. For instance to determine

³ Note that, other approaches used in the literature, i.e. ANN and regression type models, are not able to find any benchmarks for inefficient DMUs.

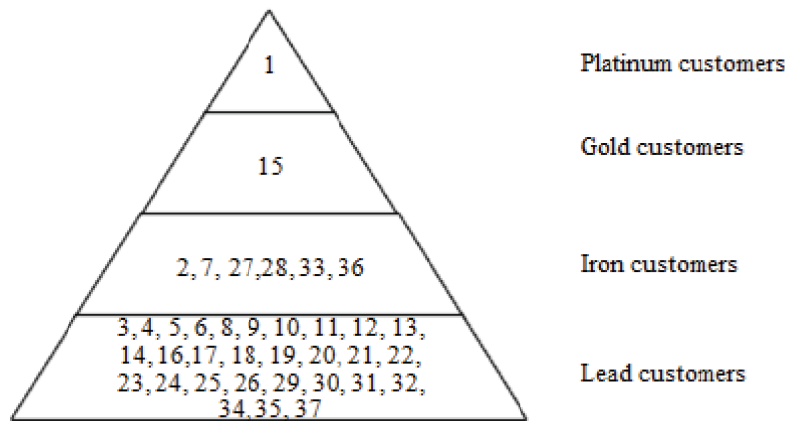


FIGURE 4. Customer value pyramid

number of Iron customers, we have $15\% \times 37 \cong 6$, where 15% is the percentage determined by Van Raaij [33] and 37 is the total number of customers. These 6 customers were classified in the customer value pyramid after Platinum and Gold customers.

We should pay particular attention to the needs of Platinum (customer #1) and Gold customers (customer #15) relative to Iron and Lead customers. Therefore, the majority of marketing resources should be allocated to them.

4. Concluding remarks. The rapid changes in the field of economical activities and agencies' involvement in competition, forces them to apply those tools and approaches which are capable of producing more competitive advantage for them and let them protect and enhance their market share. Understanding the needs and expectations of the customers and grouping them into classes with purpose of improving the efficiency of the marketing strategies is one of these approaches. In order to allocate marketing resources among customers in an optimal way, it is necessary to classify them and find good and bad customers.

In this paper, we used DEA as a multiple-criteria decision making tool to evaluate customers instead of a single criterion such as profitability of customers. AHP has been used to find the upper and lower bounds of weights of criteria. In order to have a complete ranking of customers, the cross-efficiency score of customers is used. After identifying efficient and inefficient customers, they have been classified into four groups labeled as Platinum, Gold, Iron, and Lead. After determining different types of customers, it is necessary to focus on requirements and expectations of Platinum and Gold customers relative to Iron and Lead ones.

The problem considered in this study is at initial stage of investigation and further researches can be done based on the results of this paper.

- Similar research can be repeated in the presence of imprecise data, stochastic data, and generally speaking, evaluating the value of customers under uncertainty.

- Developing a model that not only can rank customers, but also can optimally allocate marketing budgets among them.
- Some authors such as Prahalad [24] focus on the “bottom of the pyramid (BOP)” instead of “top of the pyramid (TOP)” and describe the untapped potential of the BOP. Their perception is that individually the customers on the BOP are not profitable but together they represent massive purchasing power. Therefore, the question here is which tier of the customer value pyramid is really optimal to focus on it.

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Appendix.

Table 1. The nomenclatures

DMU_d :	the decision making unit under investigation
$j=1, \dots, n$	collection of DMUs (customers)
$r=1, \dots, k$	the set of desirable outputs
$i=1, \dots, m$	the set of inputs
$s=k+1, \dots, p$	the set of undesirable outputs
y_{rd} :	r th output of DMU_d
y_{rd}^g :	r th desirable output of the DMU_d
y_{rj} :	r th output of DMU_j
y_{rj}^g :	the r th desirable output of DMU_j
μ_r^g :	the weight for r th desirable output
x_{id} :	i th input of the DMU_d
x_{ij} :	the i th input of DMU_j
v_i :	the weight for i th input
y_{sd}^b :	s th undesirable output of the DMU_d
y_{sj}^b :	the s th undesirable output of DMU_j
μ_s^b :	the weight for s th undesirable output
θ :	efficiency measure for DMU_d
s_r^+ :	shortages in r th desirable output
s_i^- :	excesses in i th input
s_s^+ :	excesses in s th undesirable output
λ_j :	reference weights (benchmarks) associated with DMU_j
w_d :	the variable which determines “return to scale” of DMU_d
\bar{y}_{sj}^b :	the y_{sj}^b which is translated into desirable output
α_i :	the lower bound of relative weight restrictions of the inputs
β_i :	the upper bound of relative weight restrictions of the inputs
ϕ_i :	the lower bound of relative weight restrictions of the undesirable outputs
ψ_i :	the upper bound of relative weight restrictions of the undesirable outputs
τ_r :	the lower bound of relative weight restrictions of the desirable outputs
γ_r :	the upper bound of relative weight restrictions of the desirable outputs
E_{dj} :	shows the relative efficiency of DMU_j with optimal weights for inputs and outputs of DMU_d
E_{dd} :	the efficiency score of DMU_d by its own optimal weights

Table 2. Data set for hypothetical numerical example

DMU	x	y^g	y^b	Efficiency scores ($v = 15$)	Efficiency scores ($v = 20$)
1	9	12	10	1.121	1.075
2	8	14	11	1	1
3	5	13	12	1	1
4	4	12	5	1	1
5	2.5	10	7	1	1
6	3	9	3	1	1
7	7	5	5	1.200	1.033
8	11	4	4	1.090	1.062
9	13	3	8	1.714	1.416
10	2	1	7	1	1

Table 3. Judgment scores for the importance of criteria using AHP

Verbal Judgments	Numerical rating
Extremely important	9
Very strongly to Extremely important	8
Very strongly important	7
Strongly to very strongly important	6
Strongly important	5
Moderately to strongly important	4
Moderately important	3
Equally to moderately important	2
Equally important	1

Table 4. Measures for evaluation of customers

x_1 : <i>Average payment period</i> ; y_1^g : <i>Credit of customer (in terms of US dollar)</i> ; y_2^g : <i>Profitability of customer (in terms of US dollar)</i> ; y_3^g : <i>Payments on due date</i> ⁴ ; y_1^b : <i>Purchase return</i> ⁵ ;

⁴This variable is a qualitative criterion. Assume that for this qualitative variable each customer is rated on a 5-point scale, where the particular point on the scale is chosen through a consensus on the part of executives within the organization. 5-point scales are common for evaluating in terms of qualitative data, and are often accompanied by interpretations such as: 1 = very bad, 2 = bad, 3 = medium, 4 = good, 5 = very good, which are easily understood by decision maker.

⁵Amounts of products that the wholesaler (customer) is not able to sell and as a result the products are returned to the factory.

Table 5. Related attributes for 37 customers

DMUs	x_1 (days)	y_1^g (\$)	y_2^g (\$)	y_3^g	y_1^b (\$)
1	7.12	4,500,000	53,598	3	152
2	6.11	5,800,000	54,458	5	242
3	15.00	5,000,000	43,152	5	2,661
4	60.31	5,000,000	41,563	3	1,900
5	75.00	5,000,000	35,033	3	2,678
6	39.90	2,500,000	27,668	3	1,347
7	7.11	3,000,000	40,987	5	234
8	25.00	5,000,000	22,242	5	2,473
9	75.40	2,650,000	39,879	1	543
10	10.34	5,000,000	18,724	4	1,481
11	43.40	3,200,000	17,398	3	1,107
12	44.78	4,000,000	17,362	3	137
13	29.56	2,700,000	19,000	1	1,234
14	14.94	3,100,000	22,000	5	1,181
15	5.80	5,000,000	50,876	5	211
16	74.32	3,700,000	33,000	1	456
17	75.48	2,700,000	31,000	3	987
18	15.00	7,000,000	23,000	5	1,283
19	44.12	5,000,000	41,000	1	1,916
20	32.70	4,500,000	47,981	3	1,327
21	59.70	2,000,000	23,400	4	1,926
22	44.31	2,500,000	19,800	1	316
23	10.71	3,900,000	16,400	5	2,001
24	47.10	5,200,000	18,300	2	927
25	15.00	4,700,000	17,546	3	949
26	28.40	3,500,000	19,800	5	1,324
27	4.74	4,500,000	34,000	5	943
28	28.70	6,500,000	27,600	2	162
29	45.30	3,000,000	29,683	1	104.33
30	60.00	4,000,000	16,987	1	773
31	31.50	4,500,000	17,890	3	965
32	80.30	6,000,000	31,400	1	1,147
33	11.30	5,200,000	18,700	5	111
34	30.00	3,500,000	16,745	3	1,492
35	57.90	2,200,000	16,547	1	1,345
36	14.67	3,700,000	18,978	5	688
37	77.80	3,400,000	21,988	3	595

Table 6. Normalized inputs and outputs

DMUs	x_1	y_1^g	y_2^g	y_3^g	y_1^b
1	0.00528	0.02948	0.05126	0.02564	0.00387
2	0.00453	0.03800	0.05208	0.04274	0.00615
3	0.01112	0.03275	0.04127	0.04274	0.06768
4	0.04471	0.03275	0.03975	0.02564	0.04832
5	0.05560	0.03275	0.03350	0.02564	0.06811
6	0.02958	0.01638	0.02646	0.02564	0.03426
7	0.00527	0.01965	0.03920	0.04274	0.00595
8	0.01853	0.03275	0.02127	0.04274	0.06289
9	0.05589	0.01736	0.03814	0.00855	0.01381
10	0.00766	0.03275	0.01791	0.03419	0.03766
11	0.03217	0.02096	0.01664	0.02564	0.02814
12	0.03319	0.02620	0.01660	0.02564	0.00347
13	0.02191	0.01769	0.01817	0.00855	0.03139
14	0.01107	0.02031	0.02104	0.04274	0.03004
15	0.00430	0.03275	0.04865	0.04274	0.00537
16	0.05509	0.02424	0.03156	0.00855	0.01160
17	0.05595	0.01769	0.02965	0.02564	0.02510
18	0.01112	0.04586	0.02200	0.04274	0.03263
19	0.03271	0.03275	0.03921	0.00855	0.04873
20	0.02424	0.02948	0.04588	0.02564	0.03375
21	0.04426	0.01310	0.02238	0.03419	0.04898
22	0.03285	0.01638	0.01893	0.00855	0.00803
23	0.00794	0.02555	0.01568	0.04274	0.05089
24	0.03491	0.03406	0.01750	0.01709	0.02358
25	0.01112	0.03079	0.01678	0.02564	0.02415
26	0.02105	0.02293	0.01893	0.04274	0.03367
27	0.00351	0.02948	0.03251	0.04274	0.02398
28	0.02128	0.04258	0.02639	0.01709	0.00413
29	0.03358	0.01965	0.02839	0.00855	0.00265
30	0.04448	0.02620	0.01624	0.00855	0.01965
31	0.02335	0.02948	0.01711	0.02564	0.02456
32	0.05953	0.03931	0.03003	0.00855	0.02917
33	0.00838	0.03406	0.01788	0.04274	0.00283
34	0.02224	0.02293	0.01601	0.02564	0.03795
35	0.04292	0.01441	0.01582	0.00855	0.03421
36	0.01087	0.02424	0.01815	0.04274	0.01751
37	0.05767	0.02227	0.02103	0.02564	0.01513

Table 7. Pairwise comparison matrix for outputs determined by DM #1

DM#1	Credit	Profitability	Payments on due date
Credit	1	4	3
Profitability		1	1
Payments on due date	1/3	1	1

Table 8. Pairwise comparison matrix for outputs determined by DM #2

DM#2	Credit	Profitability	Payments on due date
Credit	1	1/2	5
Profitability	2	1	7
Payments on due date	1/5	1/7	1

Table 9. Pairwise comparison matrix for outputs determined by DM #3

DM#3	Credit	Profitability	Payments on due date
Credit	1	3	5
Profitability	1/3	1	2
Payments on due date	1/5	1/2	1

Table 10. Pairwise comparison matrix for outputs determined by DM #4

DM#4	Credit	Profitability	Payments on due date
Credit	1	1/2	3
Profitability	2	1	3
Payments on due date	1/3	1/3	1

Table 11. Pairwise comparison matrix for input & undesirable output determined by DM #1

DM1	Purchase return	Payment period
Purchase return	1	2
Payment period	1/2	1

Table 12. Pairwise comparison matrix for input & undesirable output determined by DM #2

DM2	Purchase return	Payment period
Purchase return	1	1/2
Payment period	2	1

Table 13. Pairwise comparison matrix for input & undesirable output determined by DM #3

DM3	Purchase return	Payment period
Purchase return	1	1
Payment period	1	1

Table 14. Pairwise comparison matrix for input & undesirable output determined by DM #4

DM4	Purchase return	Payment period
Purchase return	1	1/2
Payment period	2	1

Table 15. Derived weights of outputs for each DM's opinion and inconsistency ratios

	Credit	Profitability	Payments on due date	Inconsistency ratio
DM1	0.634	0.174	0.192	0.01
DM2	0.333	0.592	0.075	0.01
DM3	0.648	0.23	0.122	0
DM4	0.333	0.528	0.14	0.05

Table 16. Derived weights of inputs for each DM's opinion and inconsistency ratios

	Purchase return	Average of payment period	Inconsistency ratio
DM1	0.333	0.667	0
DM2	0.667	0.333	0
DM3	0.5	0.5	0
DM4	0.25	0.75	0

Table 17. Results of evaluation via different approaches

DMUs	Profitability of customer (in terms of US dollar)	Result of Model (10)	Benchmarks derived by Model (11)	Cross-efficiency score
1	53,598	1.00000	1	0.98169
2	54,458	1.00000	2	0.87161
3	43,152	0.15536	15	0.12561
4	41,563	0.10237	1, 15	0.09677
5	35,033	0.07801	1, 15	0.07269
6	27,668	0.15027	1, 15	0.13476
7	40,987	0.85269	1, 15	0.78526
8	22,242	0.13977	15	0.11622
9	39,879	0.17340	1, 33	0.12746
10	18,724	0.26368	15	0.21221
11	17,398	0.15445	1, 15	0.14234
12	17,362	0.38693	33	0.25853
13	19,000	0.18575	15	0.16097
14	22,000	0.26773	15	0.22121
15	50,876	1.00000	15	0.94715
16	33,000	0.18771	1, 33	0.13631
17	31,000	0.12859	1, 33	0.10696
18	23,000	0.27301	2, 15	0.22030
19	41,000	0.12238	15	0.11077
20	47,981	0.16989	15	0.15589
21	23,400	0.10239	1, 15	0.09153
22	19,800	0.29628	33	0.21326
23	16,400	0.20923	15	0.16380
24	18,300	0.15985	1, 33	0.15110
25	17,546	0.30114	15	0.25762
26	19,800	0.18437	15	0.16207
27	34,000	0.45065	15	0.35798
28	27,600	0.55995	1, 2	0.37853
29	29,683	0.40621	33	0.25995
30	16,987	0.16318	1, 33	0.13563
31	17,890	0.19820	1, 15	0.18349
32	31,400	0.11501	1, 2	0.10215
33	18,700	1.00000	33	0.81730
34	16,745	0.16948	15	0.14639
35	16,547	0.11963	1, 15	0.10841
36	18,978	0.35592	15	0.31334
37	21,988	0.16369	33	0.12273

Table 18. Optimal values derived by Model (14)

DMUs	v_1	μ_1^g	μ_2^g	μ_3^g	μ_1^b	W
1	0.25227	0	0	0	0.75197	-0.00424
2	0.67006	0.06616	0.01985	0.01323	0.33503	-0.00098
3	0.68727	0	0	0	0.34363	-0.00480
4	0.63410	0	0	0	0.41427	-0.00495
5	0.64382	0	0	0	0.42063	-0.00503
6	0.62446	0	0	0	0.40799	-0.00488
7	0.60821	0	0	0	0.39736	-0.00475
8	0.68964	0	0	0	0.34482	-0.00482
9	0.25623	0.00010	0.00018	0.00002	0.76870	-0.00432
10	0.67865	0	0	0	0.33933	-0.00474
11	0.62392	0	0	0	0.40762	-0.00487
12	0.25276	0	0	0	0.75827	-0.00426
13	0.68381	0	0	0	0.34190	-0.00478
14	0.67847	0	0	0	0.33923	-0.00474
15	0.66979	0	0	0	0.33489	-0.00468
16	0.25575	0.00010	0.00018	0.00002	0.76724	-0.00431
17	0.25974	0	0	0	0.77423	-0.00437
18	0.67909	0.06705	0.02012	0.01341	0.33954	-0.00100
19	0.69304	0	0	0	0.34652	-0.00484
20	0.68546	0	0	0	0.34273	-0.00479
21	0.63408	0	0	0	0.41427	-0.00495
22	0.25361	0	0	0	0.76083	-0.00428
23	0.68184	0	0	0	0.34092	-0.00476
24	0.25803	0	0	0	0.76912	-0.00434
25	0.67714	0	0	0	0.33857	-0.00473
26	0.68394	0	0	0	0.34197	-0.00478
27	0.67363	0	0	0	0.33681	-0.00471
28	0.25212	0.12600	0.03780	0.02520	0.75637	0.00204
29	0.25262	0	0	0	0.75787	-0.00426
30	0.25788	0	0	0	0.76869	-0.00434
31	0.61960	0	0	0	0.40481	-0.00484
32	0.25954	0.12966	0.03890	0.02593	0.77862	0.00210
33	0.25106	0	0	0	0.75318	-0.00424
34	0.68550	0	0	0	0.34275	-0.00479
35	0.62970	0	0	0	0.41140	-0.00492
36	0.67550	0	0	0	0.33775	-0.00472
37	0.25661	0	0	0	0.76983	-0.00433

Table 19. Matrix of cross-efficiency

DMUs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.00000	0.88728	0.96812	1.00000	1.00000	1.00000	1.00000	0.96812	1.00000	0.96812	1.00000	0.96812	0.96812	0.96812	0.96812	1.00000	1.00000	0.88728
2	0.73553	1.00000	0.91847*	0.91348	0.91348	0.91348	0.91348	0.91847	0.73521	0.91847	0.91348	0.91847	0.91847	0.91847	0.91847	0.73521	0.73553	1.00000
3	0.07900	0.15050	0.15536	0.14110	0.14110	0.14110	0.14110	0.15536	0.14110	0.15536	0.14110	0.15536	0.15536	0.15536	0.15536	0.07885	0.07900	0.15050
4	0.08909	0.09269	0.10142	0.10237	0.10237	0.10237	0.10237	0.10142	0.08901	0.10237	0.10237	0.10237	0.10237	0.10237	0.10237	0.08901	0.08909	0.09269
5	0.06502	0.06914	0.07791	0.07801	0.07801	0.07801	0.07801	0.07791	0.06594	0.07791	0.07801	0.07791	0.07791	0.07791	0.07791	0.06494	0.06502	0.06914
6	0.12768	0.09363	0.14954	0.15027	0.15027	0.15027	0.15027	0.14954	0.12744	0.15027	0.15027	0.15027	0.15027	0.15027	0.14954	0.12744	0.12768	0.09363
7	0.73093	0.65634	0.84718	0.85269	0.85269	0.85269	0.85269	0.84718	0.73007	0.84718	0.85269	0.84718	0.84718	0.84718	0.84718	0.73007	0.73093	0.65634
8	0.08163	0.12354	0.13977	0.13977	0.13977	0.13977	0.13977	0.13977	0.08143	0.13977	0.13977	0.13977	0.13977	0.13977	0.08143	0.08163	0.12354	0.12354
9	0.17326	0.07132	0.11123	0.12029	0.12029	0.12029	0.12029	0.11123	0.17340	0.11123	0.12029	0.17335	0.11123	0.11123	0.11123	0.17340	0.17326	0.07132
10	0.14023	0.22294	0.26368	0.24201	0.24201	0.24201	0.26368	0.13983	0.26368	0.26368	0.24201	0.13984	0.26368	0.26368	0.26368	0.13983	0.14023	0.22294
11	0.14490	0.09807	0.15106	0.15445	0.15445	0.15445	0.15445	0.15106	0.14463	0.15106	0.15445	0.14470	0.15106	0.15106	0.15106	0.14463	0.14490	0.09807
12	0.38627	0.14463	0.20000	0.22022	0.22022	0.22022	0.22022	0.20000	0.38681	0.20000	0.22022	0.38693	0.20000	0.20000	0.20000	0.38681	0.38627	0.14463
13	0.14562	0.10424	0.18575	0.18408	0.18408	0.18408	0.18408	0.18575	0.14526	0.18575	0.18408	0.14533	0.18575	0.18575	0.18575	0.14526	0.14562	0.10424
14	0.16713	0.18930	0.26773	0.25438	0.25438	0.25438	0.26773	0.16669	0.26773	0.25438	0.25438	0.16672	0.26773	0.26773	0.26773	0.16669	0.16713	0.18930
15	0.82908	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.82757	0.82757	1.00000	1.00000	0.82656	1.00000	1.00000	1.00000	0.82757	0.82908	1.00000
16	0.18753	0.08151	0.11472	0.12460	0.12460	0.12460	0.11472	0.18771	0.11472	0.11472	0.12460	0.18767	0.11472	0.11472	0.11472	0.18771	0.18753	0.08151
17	0.12859	0.06712	0.10197	0.10793	0.10793	0.10793	0.10197	0.12854	0.10197	0.10197	0.10793	0.12853	0.10197	0.10197	0.10197	0.12854	0.12859	0.06712
18	0.15515	0.27301	0.25460	0.24072	0.24072	0.24072	0.25460	0.15482	0.25460	0.25460	0.24072	0.15476	0.25460	0.25460	0.25460	0.15482	0.15515	0.27301
19	0.09449	0.10565	0.12238	0.12097	0.12097	0.12097	0.12238	0.09436	0.09436	0.12238	0.12097	0.09430	0.12238	0.12238	0.12238	0.09436	0.09449	0.10565
20	0.13470	0.15183	0.16989	0.16868	0.16868	0.16868	0.16989	0.13456	0.16868	0.16868	0.16989	0.13443	0.16989	0.16989	0.16989	0.13456	0.13470	0.15183
21	0.08838	0.05960	0.10160	0.10239	0.10239	0.10239	0.10160	0.08820	0.10160	0.10160	0.10239	0.08823	0.10160	0.10160	0.10160	0.08820	0.08838	0.05960
22	0.29612	0.10343	0.18948	0.20497	0.20497	0.20497	0.18948	0.29612	0.18948	0.18948	0.20497	0.29628	0.18948	0.18948	0.18948	0.29612	0.29612	0.10343
23	0.10534	0.15868	0.20923	0.18958	0.18958	0.18958	0.20923	0.10501	0.20923	0.20923	0.18958	0.10504	0.20923	0.20923	0.20923	0.10501	0.10534	0.15868
24	0.15985	0.12174	0.14957	0.15519	0.15519	0.15519	0.14957	0.15966	0.14957	0.14957	0.15519	0.15968	0.14957	0.14957	0.14957	0.15966	0.15985	0.12174
25	0.20234	0.23754	0.30114	0.29030	0.29030	0.29030	0.30114	0.20183	0.29030	0.30114	0.29030	0.20187	0.30114	0.30114	0.30114	0.20183	0.20234	0.23754
26	0.13850	0.13554	0.18437	0.18139	0.18139	0.18139	0.18437	0.13818	0.18437	0.18437	0.18139	0.13821	0.18437	0.18437	0.18437	0.13818	0.13850	0.13554
27	0.22424	0.39897	0.45065	0.40718	0.40718	0.40718	0.45065	0.22270	0.45065	0.45065	0.40718	0.22359	0.45065	0.45065	0.45065	0.22270	0.22424	0.39897
28	0.50060	0.29084	0.29921	0.32564	0.32564	0.32564	0.29921	0.50104	0.29921	0.29921	0.32564	0.50104	0.29921	0.29921	0.29921	0.50104	0.50060	0.29084
29	0.40540	0.12652	0.20011	0.22113	0.22113	0.22113	0.20011	0.40619	0.20011	0.20011	0.22113	0.40621	0.20011	0.20011	0.20011	0.40619	0.40540	0.12652
30	0.16318	0.08660	0.12863	0.13623	0.13623	0.13623	0.12863	0.16304	0.13623	0.13623	0.13623	0.16311	0.12863	0.12863	0.12863	0.16304	0.16318	0.08660
31	0.17415	0.15128	0.19604	0.19820	0.19820	0.19820	0.19604	0.17383	0.19820	0.19604	0.19820	0.17386	0.19604	0.19604	0.19604	0.17383	0.17415	0.15128
32	0.11480	0.08643	0.09425	0.09936	0.09936	0.09936	0.09425	0.11478	0.09936	0.09936	0.09936	0.11473	0.09425	0.09425	0.09425	0.11478	0.11480	0.08643
33	1.00000	0.63327	0.71312	0.76336	0.76336	0.76336	0.71312	1.00000	0.71312	0.71312	0.76336	1.00000	0.71312	0.71312	0.71312	1.00000	1.00000	0.63327
34	0.12423	0.11430	0.16948	0.16602	0.16602	0.16602	0.16948	0.12391	0.16602	0.16602	0.16602	0.12396	0.16948	0.16948	0.16948	0.12391	0.12423	0.11430
35	0.11606	0.05875	0.11637	0.11963	0.11963	0.11963	0.11637	0.11583	0.11963	0.11637	0.11963	0.11591	0.11637	0.11637	0.11637	0.11583	0.11606	0.05875
36	0.26665	0.26707	0.35592	0.35000	0.35000	0.35000	0.35592	0.26603	0.35000	0.35000	0.35000	0.26609	0.35592	0.35592	0.35592	0.26603	0.26665	0.26707
37	0.16362	0.07349	0.10707	0.11559	0.11559	0.11559	0.10707	0.16366	0.10707	0.10707	0.11559	0.16369	0.10707	0.10707	0.10707	0.16366	0.16362	0.07349

*0.91847 represents the cross-efficiency score of customer 2 with the optimal weights of customer # 3
 $(0.03800 \times 0 + 0.05208 \times 0 + 0.04274 \times 0 - 0.00480)/(0.00453 \times 0.68727 + 0.00615 \times 0.34363) = 0.91847$

** Bold numbers in the leading diagonal are the simple efficiencies derived by Model (10)

Table 20. Ranking of DMUs based on different approaches

DMUs	Profitability ranking	Cross-efficiency Ranking
1	2	1
2	1	3
3	5	28
4	6	35
5	10	37
6	16	26
7	8	5
8	20	30
9	9	27
10	27	15
11	32	23
12	33	10
13	25	19
14	21	12
15	3	2
16	12	24
17	14	33
18	19	13
19	7	31
20	4	20
21	18	36
22	23	14
23	37	17
24	29	21
25	31	11
26	23	18
27	11	7
28	17	6
29	15	9
30	34	25
31	30	16
32	13	34
33	28	4
34	35	22
35	36	32
36	26	8
37	22	29

Table 21. Correlation coefficient between scores of profitability of customers and cross-efficiency

		Profitability	Cross-efficiency
Spearman's rho	Profitability	1.000	0.026
			0.878
	Cross-efficiency	0.026	1.000
		0.878	

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Received March 2010; 1st revision October 2010; final revision March 2011.

E-mail address: ma.mahdiloo@gmail.com

E-mail address: ab.noorizadeh@gmail.com

E-mail address: farzipour@yahoo.com